Section 4.6 Implicit Differentiation + Logarithmic Differentiation

Example $y = x^2$ * normal way we see it
where $y' = 2x$

Other ones may show as: $3x^2y^3 + 4xy^5 - 3x + 2y = 7$

★ 2 Main Rules to Remember: ★

The derivative of x is $\frac{d}{dx}(x) = \frac{dx}{dx} = 1$

The derivative of y with respect to x is:

$$\frac{d}{dx}y = \frac{dy}{dx} = y'$$

In a nutshell: The derivative of x WRT x is 1

The derivative of y WRT x is y'

Examples $xy + 3x^2 - 2y = 4$

Step 1 Product

$f = x$ $g = y$

$f' = 1$ $g' = y'$

2nd piece derivative $= 6x$

3rd piece derivative $= -2y'$

4th piece derivative $= 0$

Step 2 Reassemble into $f'g + g'f$ = Product Rule

$y + xy'$ ← 1st piece

Step 3 Connect pieces = $y + xy' + 6x - 2y' = 0$

Step 4 Get y' on one side:

$xy' - 2y' = -6x - y$

$y'(x - 2) = -6x - y$

negatives (too many), so multiply by (-1) to eliminate some negatives. ^_^

$y' = \frac{-6x - y}{x - 2}$ or $y' = \frac{6x + y}{2 - x}$

Example $x = \ln y$

$1 = \frac{1}{y} \cdot y'$

1st step: $\frac{1}{y}$ — 2nd step: y'

For inside piece

Final answer $y = y'$

Example $(2x-3y)^5 + 4xy^3 - \sqrt[3]{y} + \sqrt{x} = 5$

last piece $= \emptyset$

1st piece: $5(2x-3y)^4 (2-3y')$

2nd piece: Product Rule b/c multiplication
$f = 4x \quad g = y^3$
$f' = 4 \quad g' = 3y^2 \cdot y'$
$4(y^3) + (3y^2 \cdot y')(4x)$
$= 4y^3 + 12xy^2y'$

3rd piece: Power Rule $= y^{1/3}$
derivative $= \frac{1}{3}y^{-2/3} \cdot y'$

4th piece: Power Rule $= x^{1/2}$
derivative $= \frac{1}{2}x^{-1/2}$

Step 6 Combine all pieces
$5(2x-3y)^4(2-3y') + 4y^3 + 12xy^2y' - \frac{1}{3}y^{-2/3} \cdot y' + \frac{1}{2}x^{-1/2} = \emptyset$

Step 7 Simplify: $10(2x-3y)^4 - 15(2x-3y)^4 \cdot y' \Rightarrow$ 1st piece

Step 8 Recombine simplification + rest of pieces
$= 10(2x-3y)^4 - 15(2x-3y)^4 \cdot y' + 4y^3 + 12xy^2y' - \frac{1}{3}y^{-2/3}y' + \frac{1}{2}x^{-1/2} = \emptyset$

Step 9 Get y' by itself on one side and multiply whole equation by 6 to get rid of fractions

LCD $6 * (10(2x-3y)^4 - 15(2x-3y)^4 \cdot y' + 4y^3 + 12xy^2y' - \frac{1}{3}y^{-2/3}y' + \frac{1}{2}x^{-1/2} = \emptyset)$
$= 60(2x-3y)^4 - 90(2x-3y)^4 \cdot y' + 24y^3 - 2y^{-2/3}y' + 3x^{-1/2} + 72xy^2y' = \emptyset$

Step 10 Count + make sure you have all pieces of equation

Step 11 Get y' by itself, so $-90(2x-3y)^4y' - 2y^{-2/3}y' + 72xy^2y'$

, so new equation is:

$60(2x-3y)^4 + 24y^3 + 3x^{-1/2} = y'(90(2x-3y)^4 + 2y^{-2/3} - 72xy^2)$

Step 12 Now divide both sides by $\frac{1}{}$ to get y' by itself

$$\frac{60(2x-3y)^4 + 24y^3 + 3x^{-1/2}}{90(2x-3y)^4 + 2y^{-2/3} - 72xy^2} = y'$$ FINAL ANSWER

Example: $x = 3e^{2y}$

Step 1 x derivative $= 1$ } 1st piece

, so $1 = 3e^{2y}$

Step 2 $1 = 3e^{2y}$

↑ 2nd piece $= 2y'$

Step 3 $1 = 3e^{2y} \cdot 2y'$

Step 4 Get y' by itself, so

$$\frac{1}{6e^{2y}} = y'$$